$$2j \sin \Delta\phi e^{-2\eta-\Delta\eta} \int_{S_2} (E_1 \times H_1^*) \cdot da$$

$$= -2\omega\mu_0 \int_V \mathrm{Im}\,(H' \cdot H^*) dv - 2\omega\epsilon_0 \int_{V-V_f} \mathrm{Im}\,(E \cdot E'^*) dv$$

$$- 2\omega\epsilon^* \int_{V_f} \mathrm{Im}\,(E \cdot E'^*) dv + j\omega\Delta\epsilon^* \int_{V_f} E \cdot E'^* dv, \quad (14)$$

where Im denotes the imaginary part. By equating the imaginary terms in (14) and assuming small losses (*i.e.*, $\epsilon'' \ll \epsilon'$) one obtains, to first order in Δ,

$$2\Delta\phi e^{-2\eta}\, \mathrm{Re}\left[\int_{S_2} (E_1 \times H_1^*) \cdot da\right]$$

$$= \omega\Delta\epsilon' \int_{V_f} |E|^2 dv. \quad (15)$$

By making use of the relations for the output power P_{out} and the power lost in the dielectric P_{lost} given by

$$\tfrac{1}{2} e^{-2\eta}\, \mathrm{Re}\left[\int_{S_2} (E_1 \times H_1^*) \cdot da\right] = P_{\mathrm{out}},$$

and

$$\tfrac{1}{2}\omega\epsilon' \tan\delta \int_{V_f} |E|^2 dv = P_{\mathrm{lost}},$$

(15) reduces to

$$\Delta\phi = \frac{1}{2}\, \frac{\Delta\epsilon'}{\epsilon' \tan\delta}\, \frac{P_{\mathrm{lost}}}{P_{\mathrm{out}}}. \quad (16)$$

For small losses the insertion loss of the device is approximately given by

$$L \approx \frac{1}{0.23}\, \frac{P_{\mathrm{lost}}}{P_{\mathrm{out}}}\ \mathrm{db}. \quad (17)$$

Hence, by combining (16) and (17) and noting that $\epsilon' = \epsilon_0\kappa'$, one obtains the formula for the incremental phase shift

$$\Delta\phi = 0.115\, \frac{\Delta\kappa'}{\kappa' \tan\delta}\, L\ \mathrm{rad}. \quad (18)$$

A Balanced-Type Parametric Amplifier*

S. HAYASI†, ASSOCIATE MEMBER, IRE, AND T. KUROKAWA†

Summary—**A balanced-type diode amplifier is reported, in which the cutoff mode of the pumping waveguide resonating with the diode capacitances, is used as a signal circuit and a series connected diode loop is used as an idler. Theoretical noise-figure and gain-bandwidth product are derived after calculating the equivalent susceptance matrix of two diodes which are parallel-connected for the signal input and series-connected for the idler. This reveals that 1) the noise figure of the balanced-type amplifier can be expressed in the same form as that of the single diode amplifier, and 2) the gain-bandwidth product is identical to that of the single diode amplifier. In the experiment at 1900 Mc, a bandwidth of more than 200 Mc is obtained at the power gain of more than 10 db. A single-channel noise-figure of 2.5 db is measured at the pump power of 100 Mw.**

Introduction

IN ORDER TO extend the bandwidth capabilities of parametric diode amplifiers, considerable attention is presently being given to traveling-wave devices. However, in the traveling-wave-type amplifiers,[1] the operation is almost limited to the degenerative case, except a few examples,[2] and their construction is still quite complicated in order to get good phase relations between signal and idler flows. A balanced-type diode parametric amplifier was, to the best of our knowledge, suggested by Takahashi[3] but had not been tested until Kliphuis[4] made a surprising success in obtaining a bandwidth of 500 Mc in his 5300-Mc experiment.

* Received by the PGMTT, November 2, 1961; revised manuscript received January 30, 1962.

† Central Research Laboratory, Tokyo-Shibaura Electric Co., Ltd., Kawasaki, Japan.

[1] M. R. Currie, and R. W. Gould, "Coupled cavity traveling-wave parametric amplifiers, Part 1, analysis," Proc. IRE, vol. 48, pp. 1960–1973; December, 1960.
K. P. Grabowski and R. D. Weglein, "Coupled cavity traveling-wave parametric amplifiers, Part II, experimentals," Proc. IRE, vol. 48, pp. 1973–1987; December, 1960.

[2] K. P. Grabowski, "A Non-Degenerate Traveling-wave Parametric Amplifier," presented at IRE WESCON Conf., Session 40; August 22–25, 1961.

[3] H. Takahashi, "General Theory of Parametric Amplifiers," presented at Symp. S. 8, Joint Conv. of IEE and IECE of Japan, Tokyo; 1959.

[4] J. Kliphuis, "*C*-band non-degenerate parametric amplifier with 500 Mc bandwidth," Proc. IRE, vol. 49, p. 961; May, 1961.

In this paper a balanced-type diode amplifier is reported, in which the cutoff mode of the pumping waveguide resonating with the diode-capacitances is used as a signal circuit and a series connected diode loop is used as an idler. Theoretical noise-figure and gain-bandwidth product are also derived after calculating the equivalent susceptance matrix of two diodes, which are connected in parallel for signal input and connected in series for the idler.

This reveals that 1) the noise figure of the balanced-type amplifier can be expressed in the same form as that of the single diode amplifier, and 2) the gain-bandwidth product is identical to that of the single diode amplifier.

In this configuration the lead-inductances of the diodes are directly contained in signal or idler circuit, so there is no deterioration of bandwidth due to the natural resonance of the diodes. In our 1900-Mc experiment a bandwidth of more than 200 Mc is obtained at a power gain of more than 10 db. In this experiment the signal feeding circuit was a coaxial line. An experiment using a waveguide feed is under way at a higher signal frequency.

Equivalent Circuit Consideration of a Balanced-Type Parametric Amplifier

The balanced-type parametric amplifier defined here is, as shown in Fig. 1, a two-diode amplifier, in which, for signal-frequency, two diodes in parallel are resonant with an inductance and for idler frequency, a loop connecting two diodes in series is resonant. Therefore equivalent circuits of signal and idler can be shown as in Fig. 2. As the diode part of the two circuits may be depicted in a four-port box, a schematic diagram of the amplifier containing a circulator is shown in Fig. 3, where the terminals (1, 1′) and (2, 2′) in Fig. 2 correspond to the ones in Fig. 3.

Now let us define voltage and current at the terminals (1, 1′) and (2, 2′) as V, V' and I, I' where the currents flowing into the four-port network have positive sign. As pumping power couples into the idler loop magnetically, the phases of pump voltage appearing at the diodes should be opposite when looked at from the signal terminal.

The capacitance of a diode pumped with a frequency f_3 is expressed as

$$C_t = Ce^{-jx} + C_0 + Ce^{jx}, \tag{1}$$

where $x = 2 f_3 t$ and f_3 is pump frequency.

When the variation of charge δq, associated with the voltage variation δv is expressed as

$$\delta q = C_t \delta v \tag{2}$$

the relation between charge and voltage of No. 1 diode can be written as follows:[5]

[5] H. E. Rowe, "Some general properties of nonlinear elements. II, small signal theory," Proc. IRE, vol. 46, pp. 850–860; May, 1958.

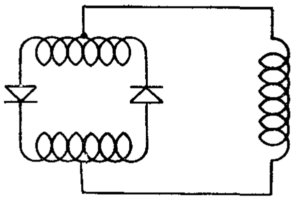

Fig. 1—An equivalent circuit of a balanced-type parametric amplifier.

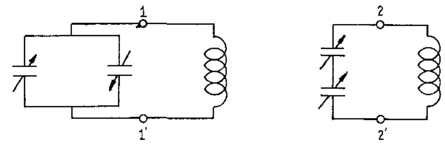

Fig. 2—Diode combinations for a signal circuit and an idler circuit. a) Signal circuit. b) Idler circuit.

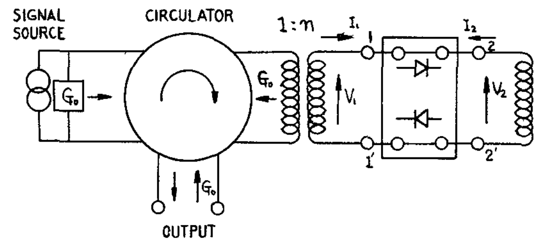

Fig. 3—Schematic diagram of the balanced-type parametric amplifier.

$$\left.\begin{aligned} Q' &= C_0'V' + C'V_2'^* \\ Q_2' &= C'V'^* + C_0'V_2' \end{aligned}\right\} \tag{3}$$

where the asterisk shows the conjugate.

At No. 2 diode, the pump phase differs by 180° from No. 1 diode, and therefore C_t has to be written as follows:

$$C_t = -C_1e^{-jx} + C_0 - C_1e^{jx}. \tag{4}$$

Substituting (3) into (2), we have the following relations:

$$\left.\begin{aligned} Q_1'' &= C_0''V_1'' - C_1''V_2''^* \\ Q_2'' &= -C_1''V_1''^* + C_0''V_2'' \end{aligned}\right\}. \tag{5}$$

As no loss of generality is incurred by assuming that the two diodes are identical, we make

$$C_0' = C_0'' = C_0, \qquad C_1' = C_1'' = C_1.$$

Since in the signal circuit two diodes are in parallel, and in the idler, series connected, voltage and charge at the signal and idler terminals have the following relation to the ones at the diode terminals.

$$\left.\begin{aligned} V_1' &= V_1'' = V_1 \\ Q_1' &+ Q_1'' = Q_1 \\ V_2' &= -V_2'' = V_2/2 \\ Q_2' &= Q_2'' = Q_2 \end{aligned}\right\}. \tag{6}$$

From (3) and (5), we have

$$\left.\begin{aligned} Q_1 &= 2C_0V_1 + C_1V_2^* \\ Q_2 &= C_1V_1^* + \frac{C_0}{2}V_2 \end{aligned}\right\}. \tag{7}$$

When $I = j\omega Q$, (7) is written as follows:

$$\left.\begin{aligned} I_1 &= j\omega_1 2C_0V_1 + j\omega_1C_1V_2^* \\ I_2 &= j\omega_2C_1V_1^* + j\omega_2\frac{C_0}{2}V_2 \end{aligned}\right\}. \tag{8}$$

This is almost the same expression as in the one-diode case, except that the diagonal coefficients of the matrix expression of the diodes differ by 2 at the signal circuit and by $\frac{1}{2}$ at the idler. This can be intuitively understood from the fact that two diodes are in parallel at the signal circuit and in series at the idler. It may be convenient to use the same notation as Greene and Sard,[6]

$$\begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} = \begin{bmatrix} 2\omega_1C_0 & \omega_1C_1 \\ \omega_2C_1 & \omega_2\dfrac{C_0}{2} \end{bmatrix}. \tag{9}$$

Noise Temperature

For the noise temperature of the balanced-type amplifier, the general form obtained by Greene and Sard can still be useful.

$$[Te]_0 \cong \frac{G_1}{G_g}T_1 + \frac{f_{10}}{f_{20}}\left(1 + \frac{G_1}{G_g}\right)T_2. \tag{10}$$

All notations in (10) are same as Greene and Sard's paper except conductances G_1, G_2, and temperature T_1, T_2 of signal and idler circuits which are defined as

$$\left.\begin{aligned} G_1 &= 2G_{D1} \\ G_2 &= \frac{G_{D2}}{2} + G_i \end{aligned}\right\} \tag{11}$$

and

$$\left.\begin{aligned} T_1 &= T_D \\ T_2 &= \frac{\frac{1}{2}G_{D2}T_D + G_iT_i}{\frac{1}{2}G_{D2} + G_i} \end{aligned}\right\}. \tag{12}$$

If the definition of x and Z appearing in (13) of the referenced paper are chanted as follows:

$$\left.\begin{aligned} x &= 1 + \frac{G_g}{2G_{D1}} \\ Z &= 1 + 2\frac{G_i}{G_{D2}} \end{aligned}\right\}, \tag{13}$$

then we have exactly the same expression as in the previous paper,

$$\frac{[T_e]_0}{T_D} \cong \left(\frac{x}{x-1}\right)\left[1 + \frac{1}{y}\left(t + \frac{1-t}{Z}\right)\right] - 1. \tag{14}$$

[6] J. C. Greene and E. W. Sard, "Optimum noise and gain-bandwidth performance for a practical one-port parametric amplifier," Proc. IRE vol. 48, pp. 1583–1590; September, 1960.

The optimum condition of (14) is

$$x_{\text{opt}} = 1 + \sqrt{1 + \frac{1}{r}} \tag{15}$$

or

$$y_{\text{opt}} = \sqrt{1 + \frac{1}{r}} - 1. \tag{16}$$

From (16) the optimum pumping frequency f_3 can be obtained as

$$\left[\frac{f_3}{f_{10}}\right]_{\text{opt}} = \sqrt{1 + \frac{1}{r}}. \tag{17}$$

Eq. (17) can be written in much simpler form provided the usual varactors are employed in the amplifier. The following assumptions are not limited to the special case:

$C_1/C_0 = \frac{1}{3}$ and $f_c = 70$ kMc at a usual bias voltage.

In the case when signal frequency is 1.9 kMc, r is calculated as

$$r = \left(\frac{f_{10}}{f_c}\cdot\frac{C_0}{C_1}\right)^2 = (0.0815)^2. \tag{18}$$

This tells that the right hand term of (17) can be surely expressed by $\sqrt{1/r}$ only, so we have

$$\left[\frac{f_3}{f_{10}}\right]_{\text{opt}} = \frac{f_0}{f_{10}}\frac{C_1}{C_0}. \tag{19}$$

At this optimum condition we have the optimum value of noise temperature as follows:

$$\begin{aligned} \left[\frac{[T_e]_0}{T_D}\right]_{\text{opt}} &\cong 2r\left[1 + \sqrt{1 + \frac{1}{r}}\right] \\ &\cong 2\sqrt{r} \\ &= 2\frac{f_{10}}{f_c}\frac{C_0}{C_1}. \end{aligned} \tag{20}$$

As a conclusion it can be stated that the optimum noise temperature of the balanced-type parametric amplifier is the same as the one-diode amplifier, but the optimum condition is expressed in a slightly different form.

Maximum Gain-Bandwidth Product

Gain-bandwidth product of the balanced-type amplifier is also expressed in the same form as the one-diode amplifier. That is,

$$\sqrt{K_0}\frac{\beta}{f_{10}} \cong \left[\frac{2}{1 + \dfrac{G_1}{G_g}}\right]\left[\frac{1}{f_{10}\left(\dfrac{1}{\beta_1} + \dfrac{1}{\beta_2}\right)}\right], \tag{21}$$

where K_0 is power gain at the midband, and β, β_1 and β_2 are, respectively, the bandwidth of the over-all amplifier, the signal circuit, and the idler dircuit.

At first sight, β_1 and β_2 appear to be slightly different from the one-diode case. However, a careful calculation shows they are identical with the one-diode case, where the signal and idler circuits are made by simple resonances of the diode, respectively. Thus, we have

$$\left.\begin{aligned} \beta_1 &= \frac{xf_{10}^2}{f_c} \\ \beta_2 &= \frac{zf_{20}^2}{f_c} \end{aligned}\right\}. \tag{22}$$

The optimum value of the gain-bandwidth product and the corresponding optimum value of x become exactly the same as those of Greene and Sard. They are

$$\left[\sqrt{K_0}\,\frac{\beta}{f_{10}}\right]_{\text{opt}} \cong \frac{f_{10}}{f_c}\left[\sqrt{1+\frac{y}{r}}-1\right] \tag{23}$$

and

$$x_{\text{opt}} = 1 + \sqrt{1+\frac{y}{r}} \tag{24}$$

or

$$z_{\text{opt}} = \frac{\sqrt{1+\frac{y}{r}}-1}{y^2}. \tag{25}$$

In the usual case

$$y/r \gg 1;$$

so (23) takes much simpler form as

$$\left[\sqrt{K_0}\,\frac{\beta}{f_{10}}\right]_{\text{opt}} \cong \sqrt{\frac{f_{20}}{f_{10}}}\,\frac{C_1}{C_0}. \tag{26}$$

This means that in the case where the cutoff frequency of the diode is fairly large, the gain-bandwidth product depends linearly on the pump excitation ratio C_1/C_0 and not on the cutoff frequency.

Structure of the Amplifier

As seen in Fig. 4 and Fig. 5, two diodes are aligned longitudinally in the center of the pump waveguide. One set of diode ends are connected with each other at the signal terminal which is also connected to the waveguide by a fine wire; the other set of diode ends are passed to the waveguide by by-pass condensers. Looking from the signal terminal, two diodes are arranged in same polarity to each other, therefore they are both in the opposite polarity along the loop connecting the two diodes. At signal frequency, the dimensions of the pump waveguide are such that it is beyond cutoff and no signal power can flow longitudinally. Only transverse current flows, which is just like a Lecher wire shorted at both ends. As shown in Fig. 6, diodes connected at the

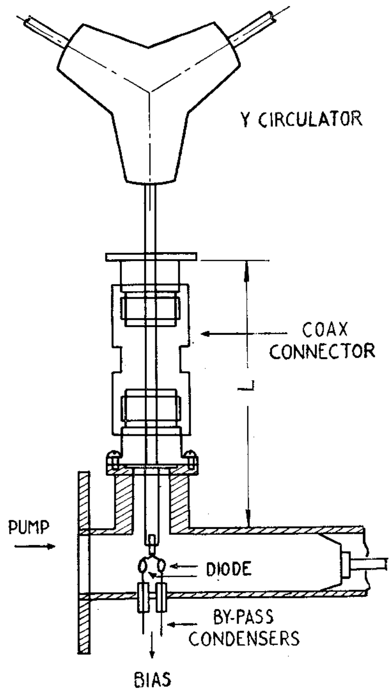

Fig. 4—Details of the amplifier construction.

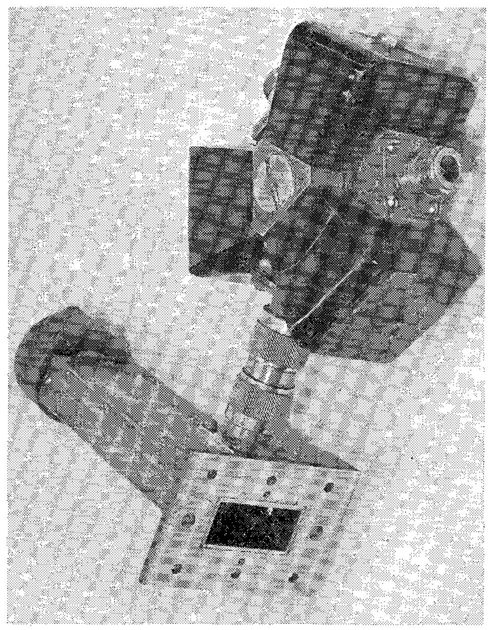

Fig. 5—L-band balanced-type parametric amplifier.

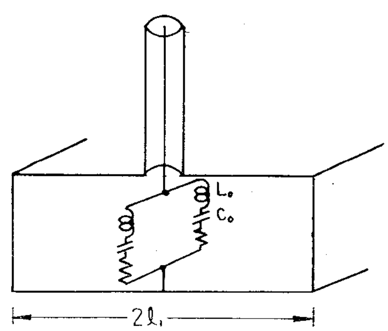

Fig. 6—Schematic diagram of a diode mounting in the pump waveguide.

center of the waveguide can be resonated with the waveguide transversely for the signal frequency. The diode loop is placed in the maximum E plane, which achieves good coupling with pump power, especially when the end of the pump waveguide is shorted at a distance of $\lambda g/2$ from the diode loop. Looking at the signal terminal, the polarities of induced voltage at the two diodes are opposite for pump frequency and the same for signal frequency, so the corresponding idler current should be opposite at the two diodes. This means the idler current is circulating along the diode loop.

Denote inductance of the diode L_0, inductance of a fine wire connecting the diodes to the waveguide L_1, capacitance of the diodes C_0, and characteristic impedance of the waveguide looking transversely Z_0, and its lenght, *i.e.*, the wider dimension of the waveguide $2l_1$, then resonance frequency λ_{10} is calculated by

$$j\omega_{10}(L_0 + L_1) + \frac{1}{j\omega_{10}C_0} = -jZ_0 \tan \frac{2\pi l_1}{\lambda_{10}} \tag{27}$$

where λ_{10} is corresponding wavelength of the waveguide.

The resonance frequency ω_{10} as a function of bias voltage, which is measured at the by-pass condensers, is shown in Fig. 7. It is seen that the resonance frequency is also dependent upon how the diodes are connected to the waveguide. The resonance frequency associated with one diode is shown in the same figure. By comparing the two cases, *i.e.*, with one diode and two diodes, the natural resonance of the diode can be measured.

Let us explain the simplest case, *i.e.*, the two diodes are completely the same. Noting the inductance when looking into the waveguide transversely from the diode terminal, which includes the inductance of the connecting wire L_2 and the resonant frequencies associated with one diode and the two diodes ω_1 and ω_2 respectively, we have

$$\omega_1 = \frac{1}{\sqrt{(L_2 + L_0)C_0}}$$

$$\omega_2 = \frac{1}{\sqrt{(2L_2 + L_0)C_0}}.$$

From these two equations the natural resonant frequency of the diode ω_n is obtained as follows:

$$\omega_n = \frac{1}{\sqrt{L_0C_0}} = \frac{\omega_1\omega_2}{\sqrt{2\omega_2^2 - \omega_1^2}}. \tag{28}$$

Eq. (28) is a convenient expression for obtaining the natural resonance of the diode because there is no need for measuring the capacitance of the diode. From Fig. 8 we have $\omega_1 = 1480$ Mc and $\omega_2 = 1120$ Mc at the bias of -1 v each where one wire is used. From this measurement, the natural resonance of the diode ω_n is calculated as 3135 Mc at the bias of -1 v each. Resonant frequencies of one diode connected to the waveguide with one wire and four wires are also shown in the same figure by circles and crosses, respectively.

The resonant frequency of the idler circuit as a function of bias voltage is shown in Fig. 8. Starting from 2370 Mc at 0-bias voltage, the resonant frequency rises to 2900 Mc at the bias voltage of -2 v. The measurement beyond the bias of -2 v was very difficult because Q of the idler circuit becomes fairly low.

Titanium condensers whose outer sides are soldered to the waveguide are used for feeding a bias voltage to a diode (see Fig. 9). These two condensers are included in the idler loop and may have some effect on the lowering of its Q.

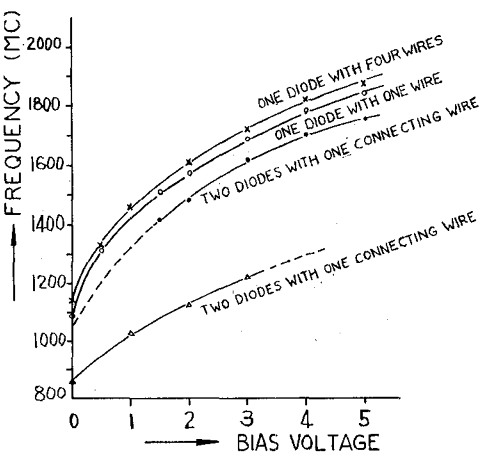

Fig. 7—Resonance frequency of a signal circuit as a function of bias voltage.

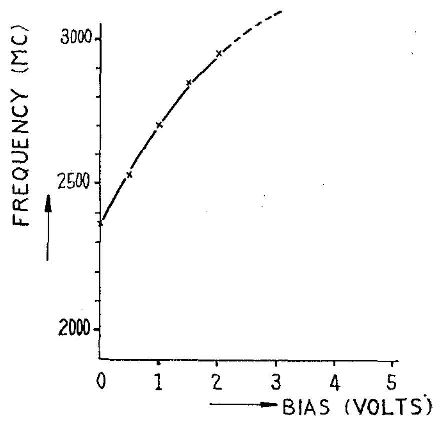

Fig. 8—A resonance of the idler circuit as changing a bias voltage.

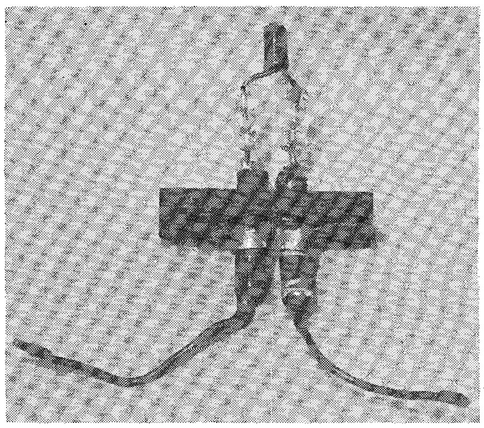

Fig. 9—Diode mounts. This also shows the idler circuit.

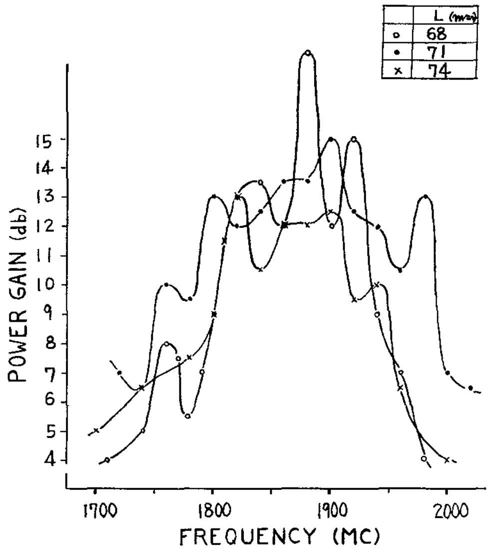

Fig. 10—Power gain of the amplifier as a parameter of the distance between the amplifier cavity and the circulator. The measurement has been done in three cases of $L=68.71$ and 74 mm.

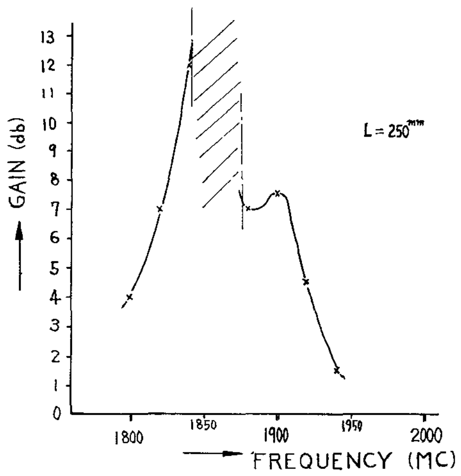

Fig. 11—Power gain of the amplifier when a low-pass filter ($fc=4100$ Mc) is inserted between the amplifier and the circulator. A region 1840–1880 Mc becomes unstable.

Experimental Results

Tests for amplification were performed with diodes whose resonance behavior vs bias voltage is shown by (X) in Fig. 7. This measurement was done with the signal terminal connected to the waveguide by four fine wires. This arrangement, however, showed very narrow amplification behavior. By taking off all of the connecting wires, a broad-band amplification is obtained. This also depends on the coaxial line length between the diodes and the circulator and a stub tuner placed at the output terminal of the Y circulator.

Amplification behavior with changing coaxial line length connecting the diodes and the circulator (68, 71, and 74 mm) is shown in Fig. 10. From these experiments it can be seen that the case of L of 71 mm is the best; a bandwidth of 220 Mc at a gain of more than 10 db is obtained. The pump frequency is 5175 Mc and the power required for the optimum condition is 100 Mw. By comparing the optimum gain-bandwidth value given by (26), the bandwidth β is calculated as 220 Mc, assuming C_1/C_0 be 0.35 and $K_0=16$ (12 db).

The case where a low-pass filter whose cutoff frequency is 4100 Mc is inserted between the circulator and the diodes is shown in Fig. 11. Here L equals 250 mm including the filter, and some instability is observed in frequency range between 1840 Mc and 1880 Mc.

Over-all noise figure and power gain as a function of pump power are shown in Fig. 12. The measurements were done for 1900 Mc and 1840 Mc where L equals 74 mm and bias voltages are -3.9 v. The optimum noise figure obtained is 2.5 db at the optimum gain condition. The noise figure increases with departure from the optimum pump power.[7] This is due to the fact that the noise figure of the following receiver, about 11 db, becomes effective in the over-all noise figure.

[7] R. C. Knechtli and R. D. Weglein, "Low-noise parametric amplifier," Proc. IRE, vol. 48, pp. 1218–1226; July, 1960.

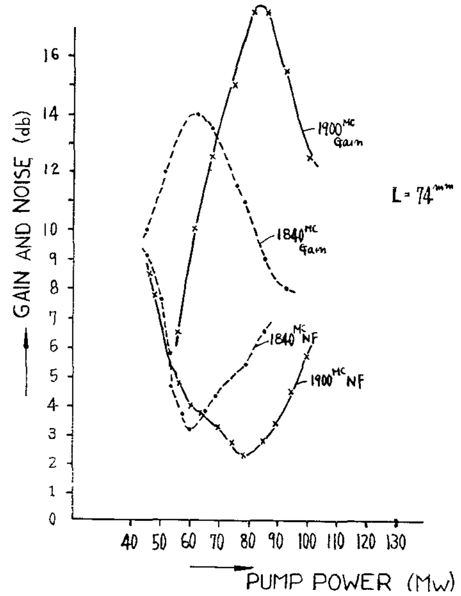

Fig. 12—Over-all power-gain and noise figure as a function of pump power.

There are many ripples in the amplifier gain produced by changing signal frequency. They seem, however, to be avoided by getting a proper match at the signal input, including the circulator.

The diodes which are used are gold-bonded Germanium and are prepared by our transistor factory. Cutoff frequency of these diodes is almost 70 kMc at a bias voltage of -2 v.

Acknowledgment

The authors are grateful to T. Tarui for his constructive criticism, especially in deriving (9). The authors are also indebted to S. Mita for his encouragement.